# Sensitive detection of radiation trapping in cold-atom clouds

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In this paper, we calculate the effect of radiation trapping on the photon statistics of the light scattered from optical molasses. We propose that an intensity correlation function measurement may be sensitive to the presence of radiation trapping at an on-resonance optical depth as low as 0.1, more than an order of magnitude less than where effects of multiple scattering in cold-atom clouds have been previously observed [T. Walker, D. Sesko, and C. Wieman, Phys. Rev. Lett. **64**, 408 (1990); D. Sesko, T. Walker, and C. Wieman, J. Opt. Soc. Am. B. **8**, 946 (1991)].

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## I. INTRODUCTION

Radiation trapping in atomic vapors refers to the reabsorption of spontaneously emitted photons, and has been studied extensively in atomic spectroscopy, astrophysics, and plasmas [1]. In the context of cold atoms, radiation trapping [2] was recognized as being principally responsible for preventing trapped atomic samples from becoming colder [3] and denser [4]. This is because an atom, upon absorbing a spontaneously emitted photon by a neighbor, experiences a momentum kick away from the emitter. In Ref. [1], the presence of radiation trapping induced abrupt changes in the cloud shape under certain conditions. These changes became observable only at number densities of $10^{10}$–$10^{11}/\mathrm{cm}^3$ and an optical depth (for an on-resonant probe) of 3. More recently, interest in radiation trapping has been revived in the context of electromagnetically induced transparency [5]. In a coherently prepared atomic sample, although the number of atoms undergoing spontaneous emission are relatively few, these spontaneous photons incoherently pump nearby atoms, destroying atomic coherence [6]. In Ref. [5], a laser field was used to create a coherent superposition of ground-state Zeeman sublevels. At densities above $5\times10^{10}/\mathrm{cm}^3$, the presence of radiation trapping was found to increase the decay rate of the Zeeman coherence.

In the context of laser-cooled atoms, a rather direct way to sensitively explore radiation trapping is to investigate the photon statistics of scattered light. Previous works have noted the importance of the frequency spectrum of the scattered light in determining the strength of repulsive radiation trapping forces [1], and in calculating the heating of the atoms owing to radiation trapping [3,7]. Further, some workers speculate that they may have seen some preliminary evidence of the effect of radiation trapping on the two-time intensity correlation function $\langle I(t)I(t+\tau)\rangle$ [8]. However, in Ref. [8], no attempt was made to include multiple scattering in the theory.

In this paper, we calculate the two-time intensity correlation function for the light scattered from optical molasses, incorporating a simple model of radiation trapping. In our model, the atoms are approximated as simple two-level systems being coherently pumped by a near-resonant laser beam that transfers some population to the excited state. The coherent pump is chosen as weak to describe the typical situation in optical molasses where the combined optical power in the multiple trapping laser beams is usually of the order of the saturation intensity, meaning that the excited state fraction seldom exceeds 10%. According to our model, besides coherent excitation by the laser, the atoms experience incoherent pumping by a thermal photon reservoir formed by the spontaneous emission from other atoms [5] in the trap. The spontaneously emitted photons are dephased and depolarized with respect to the coherent pumping field. In contrast to Ref. [5], Doppler broadening is included in our model, however, line broadening owing to the Raman transitions that would occur in real multilevel atoms is ignored [8,9]. A simple expression for the degree of second-order coherence of the scattered light is obtained, in which the contribution from reabsorption of spontaneous emission is clearly displayed. We find that the coherence properties of the scattered light are sensitive to radiation trapping even for number densities around $10^8$–$10^9/\mathrm{cm}^3$ and optical depths as low as 0.1.

Section II describes the simple physical model we use for describing radiation trapping in terms of the probability of reabsorption in the sample of a photon emitted by an atom, and how the reabsorption of fluorescent light is related to the optical depth of the atom sample. In Sec. III, we calculate the intensity correlation function for the light scattered from cold moving atoms, including the effects of radiation trapping. In Sec. III D, we plot and discuss our results.

## II. PHYSICAL MODEL OF RADIATION TRAPPING IN ATOMIC SAMPLES

Let us assume a uniform spherical distribution (diameter $l$) of two-level atoms (ground state $|g\rangle$, excited state $|e\rangle$) with number density $n$ as shown in Fig. 1. Suppose a resonant photon starts at one end and traverses through the sample. The probability that this photon is absorbed by an atom is simply $n\sigma l$ (provided $n\sigma l \ll 1$), where $\sigma$ is the on-resonance absorption cross section. $n\sigma l$ is actually the number of "collisions" suffered by a particle moving through a collection of targets with number density $n$, and in the context of the absorption of light in matter is also known as the *optical depth* because it describes the exponential attenuation experienced by a light beam propagating through the medium (see, for example, Ref. [10]).

In the context of a cold-atom cloud, the photon traversing

 

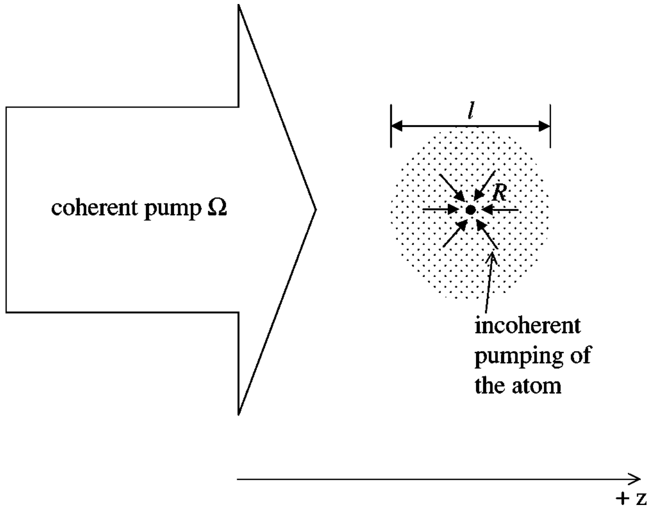


FIG. 1. Physical model of radiation trapping: An atom in the uniform cold cloud is depicted being pumped by coherent excitation from the laser as well as by a background of incoherent light. The origin of the incoherent background is the spontaneous emission from atoms in the cloud. As explained in the text, the incoherent pumping rate $R$ is given by $R = n_{th}\gamma$, where $n_{th}$=probability of reabsorption of a photon by an atom=(probability some other atom emits a photon)×(probability that this photon gets absorbed in the sample)$=\rho_{ee}\times n\sigma l$. See Sec. III for explanation of symbols.

through the sample could have come from the trapping laser itself, or have been emitted by an atom excited by the trap laser. If we denote the probability of a two-level atom being in the excited state owing to laser excitation as $\rho_{ee}$, then $\rho_{ee} = (I/2I_s)/(1 + I/I_s + 4\Delta^2/\gamma^2)$ [14], where $I$ is the trap laser intensity, $I_s$ is the saturation intensity for the transition, $\Delta$ is the laser detuning from atomic resonance, and $\gamma$ is the reciprocal of the excited state lifetime. Then, $\rho_{ee}\gamma$ is simply the number of photons emitted per second by a trapped atom as a result of being illuminated by the trap laser. These photons traverse through the cold atom cloud, with probability $n\sigma l$ of being reabsorbed (or trapped in a certain sense) in the cloud. Therefore, the probability that an atom will emit a photon, which will be eventually reabsorbed in the sample is $n\sigma l\rho_{ee}$. If we denote this probability as $n_{th}$ and the number of reabsorptions/sec per atom as $R$, then

$$n_{th} = n\sigma l\rho_{ee},$$

$$R = n_{th}\gamma. \tag{1}$$

In other words, $R$ is the incoherent pumping rate per atom due to reabsorption. One may alternatively think of the reabsorbed light as a reservoir of blackbody radiation with mean photon occupation number $n_{th}$ per mode [5,13]. In our case, $n_{th} \ll 1$, hence the above physical interpretation of $n_{th}$ as a probability is justified.

A satisfying feature of our simple physical model of radiation trapping is that $R$ depends on $\rho_{ee}\gamma$, implying that reabsorptions occur only if coherent radiation from the laser is first absorbed by the atoms and then spontaneously emitted, as should be the case. In the same vein, we see that $n_{th}$ arises from $\rho_{ee}$, and is zero if the excited state fraction is zero.

By plugging in typical numbers for optical molasses, we can see how large $n\sigma l$ and $n_{th}$ are expected to be. In the case of typical $^{85}$Rb molasses, $n = 10^8$–$10^9$/cm$^3$, $\sigma = 2.91 \times 10^{-13}\,\mathrm{m}^2$, $l \approx 1$ mm, $\gamma = (26.63\ \mathrm{ns})^{-1}$, and $I \approx 2I_s$ where $I_s = 1.64$ mW/cm$^2$. This means that $n\sigma l$ ranges from about 0.01 to 0.2, and $n_{th}$ ranges from about 0.001 to 0.02. Because $n_{th} \ll 1$, it is valid to use the physical picture of radiation trapping presented above to analyze typical molasses. The description of $n_{th}$ as a probability breaks down only when the optical depth becomes comparable to (or exceeds) unity, at that point $n\sigma l$ can no longer be interpreted as a probability.

However, our model remains a crude one because, as in Ref. [5], we do not take into account the dependence of the radiation trapping on the locations of the atoms, and the directions in which the spontaneous photons are emitted. We also ignore the frequency dependence of the atomic emission and the absorption cross section, assuming on-resonant values for both. Further, the atom density in the cloud is not uniform as has been assumed here. Hence, a quantitatively accurate prediction of $n_{th}$ is difficult. Even so, we show below that small amounts of radiation trapping are expected to significantly influence the intensity correlations of the light scattered from the cloud.

First, we use the optical Bloch equations to calculate the field emitted by a moving atom taking into account the incoherent pumping by radiation from other atoms in the cloud. We then apply the quantum regression theorem to construct the two-time correlation function for the field radiated by this atom.

## III. LIGHT SCATTERED BY THE MOVING ATOM IN THE PRESENCE OF RADIATION TRAPPING

Since we are interested in measuring the intensity correlations of the light scattered from optical molasses, we first specify the two-time correlation function that needs to be evaluated.

### A. Intensity correlation function for the scattered light

The degree of second-order temporal coherence $g^{(2)}(\tau)$ of a polarized light wave of intensity $I$ emitted by a spatially coherent thermal source is given by [11,12]

$$g^{(2)}(\tau) \equiv \frac{\langle \hat{I}(t)\hat{I}(t+\tau)\rangle}{\langle \hat{I}(t)\rangle^2} = 1 + |g^{(1)}(\tau)|^2, \tag{2}$$

where $g^{(1)}(\tau) \equiv \langle \hat{E}(t)\cdot\hat{E}^*(t+\tau)\rangle/\langle \hat{I}(t)\rangle$ is the degree of first-order coherence and, by the Weiner-Khintchine theorem, is the Fourier transform of the frequency spectrum of the light wave. $\hat{\vec{E}}(\vec{r},t)$, the operator for the field radiated by the atomic dipole, is given by $\hat{\vec{E}}(\vec{r},t) = \vec{K}(\vec{r})\hat{\sigma}_+(t)$, where $\vec{K}(\vec{r}) = (\omega^2/4\pi\epsilon_0 c^2)(\vec{d}/r - (\vec{d}\cdot\vec{r})\vec{r}/r^3)$ is the usual spatial dipole pattern at point $\vec{r}$ radiated by an electric dipole $\vec{d}$ oscillating at frequency $\omega$, and $\hat{\sigma}_{\pm}(t)$ are standard notations for the atomic raising and lowering dipole operators [11,13] in the Heisenberg picture (at $t=0$, $\hat{\sigma}_{+}\equiv|e\rangle\langle g|$ and $\hat{\sigma}_{-}\equiv|g\rangle\langle e|$). The spatial dependence of the electric field cancels out in the above expression for $g^{(1)}(\tau)$, and we obtain

$$g^{(1)}(\tau)=\frac{\langle\hat{\sigma}_{+}(t)\hat{\sigma}_{-}(t+\tau)\rangle_{ss}}{\langle\hat{\sigma}_{+}(t)\hat{\sigma}_{-}(t)\rangle_{ss}}, \tag{3}$$

where the subscript $ss$ denotes the steady state.

Note that Eq. (2) is true only for a collection of radiators that radiate independently of each other, as is the case in a chaotic source. In the absence of radiation trapping, the trapped atoms certainly act like independent radiators and Eq. (2) is valid. However, once the atoms start absorbing photons emitted by other atoms, we can no longer treat them as independent radiators. Nevertheless, we feel justified in using Eq. (2) to describe radiation trapping because in our model each atom interacts with a reservoir of incoherent photons, not with other atoms directly. While it is true that the reservoir exists only because of the spontaneous emission from all the atoms in the cloud, we may still think of each atom being pumped separately by a coherent source (the laser) and an incoherent source (the reservoir). As such each atom emits in response to this excitation independently of the other atoms. At any rate, the coupling between the atoms through reabsorption events is very weak in the case of optical molasses owing to the weak laser excitation and low atomic number densities.

Therefore, our task reduces to finding expectation values of single-time and two-time $\hat{\sigma}$ operators. We now evaluate these atomic dipole correlation functions, taking into account the motion of the atom, with the help of the optical Bloch equations.

### B. The optical Bloch equations for the moving atom

We assume the interaction energy of the atomic dipole $\vec{d}$ with the incident light field $\vec{\mathcal{E}}$ to be $-\vec{d}\cdot\vec{\mathcal{E}}$. If the excitation laser is a single beam propagating in the $+z$ direction, and a stationary atom is located at the origin, then the field incident on the atom is $\vec{\mathcal{E}}(=\frac{1}{2}\vec{\epsilon}\,\mathcal{E}_0\,e^{-i\omega t}+\text{c.c.})$. However, if the atom at the origin is moving with velocity $\vec{v}$, then the electric field at the atom is $\frac{1}{2}\vec{\epsilon}\,\mathcal{E}_0\,e^{-i(\vec{k}\cdot\vec{v}t-\omega t)}+cc$. If we define the Rabi frequency $\Omega$ as usual for a stationary atom, $\Omega\equiv\vec{d}_{eg}\cdot\vec{\epsilon}/\hbar\mathcal{E}_0$, where $\vec{d}_{eg}$ is the dipole matrix element between the ground and excited states, then for an atom located at the origin but moving with velocity $\vec{v}$ the Rabi frequency can be written as $\Omega e^{-i\vec{k}\cdot\vec{v}\,t}$. In the following, we include the effect of atomic motion on the interaction of the atom with the laser beam, but do not take into account the effect of atomic motion on the interaction of the atom with trapped photons.

In order to calculate the two-time correlation function, we start by writing down the optical Bloch equations for the atomic populations and coherence of the moving atom:

$$\begin{aligned}\dot{\rho}_{ee}&=-(n_{th}+1)\gamma\rho_{ee}+n_{th}\gamma\rho_{gg}-\left(\frac{i}{2}\Omega e^{-i\vec{k}\cdot\vec{v}t}\rho_{eg}^{*}+\text{c.c.}\right)\\&=-\dot{\rho}_{gg}\\ \dot{\rho}_{eg}&=-\left(n_{th}+\frac{1}{2}\right)\gamma\rho_{eg}-i\Delta\rho_{eg}-\frac{i}{2}\Omega e^{-i\vec{k}\cdot\vec{v}t}(\rho_{ee}-\rho_{gg})\\&=\dot{\rho}_{ge}^{*},\end{aligned} \tag{4}$$

where, in usual notation (see, for instance, Ref. [14]), $\rho_{gg}$ and $\rho_{ee}$ are the atomic populations in the ground and excited states, respectively, and $\rho_{eg}$ describes the complex amplitude of the induced atomic dipole. As usual [11], $\rho_{eg}$ and $\rho_{eg}^{*}$ are related to the expectation values of the raising and lowering atomic $\hat{\sigma}_{\pm}$ dipole operators as follows: $\rho_{eg}\equiv\langle\hat{\sigma}_{-}\rangle e^{i\omega t}$ and $\rho_{eg}^{*}\equiv\langle\hat{\sigma}_{+}\rangle e^{-i\omega t}$. Thus, the action of the raising operator $\hat{\sigma}_{+}$ (lowering operator $\hat{\sigma}_{-}$) on the ground state $|g\rangle$ yields the excited state $|e\rangle$ (0), and on the excited state $|e\rangle$ yields 0 (ground state $|g\rangle$). Also, as usual, $\rho_{ee}=\langle\sigma_{ee}\rangle$ and $\rho_{gg}=\langle\sigma_{gg}\rangle$ (at $t=0$, $\sigma_{ee}=|e\rangle\langle e|$ and $\sigma_{gg}=|g\rangle\langle g|$). The symbols $\gamma$ and $\Delta$ represent, respectively, the radiative decay rates from the excited state and the laser detuning ($\Delta\equiv\omega_{eg}-\omega$, where $\omega_{eg}$ is the atomic resonance frequency and $\omega$ is the driving laser frequency) from atomic resonance. A physical explanation of $n_{th}$ in terms of the probability of reabsorption of a fluorescent photon is given in the preceding section. One can see from Eq. (4) that the role of $n_{th}$ is to (a) incoherently pump atoms out from $|g\rangle$ to $|e\rangle$ causing an increase in the radiative decay rate from the excited state, and (b) cause an increase in the decay rate of the dipole coherence.

From Eqs. (4), it is straightforward to obtain the following solution for the dipole coherence $\rho_{eg}(t)\equiv\langle\hat{\sigma}_{-}(t)\rangle e^{i\omega t}$:

$$\begin{aligned}\langle\hat{\sigma}_{-}(t)\rangle e^{i\omega t}&=\langle\hat{\sigma}_{-}(t')\rangle e^{i\omega t'}e^{-(\gamma'/2+i\Delta)(t-t')}\\&\quad-\frac{i}{2}\Omega e^{-(\gamma' t/2+i\Delta)t}\int_{t'}^{t}dt''[\rho_{ee}(t'')\\&\quad-\rho_{gg}(t'')]e^{(\gamma'/2+i\Delta)t''}e^{-i\vec{k}\cdot\vec{v}\,t''},\end{aligned} \tag{5}$$

where $\gamma'\equiv(2n_{th}+1)\gamma$ and $t'$ is the initial time for our measurement. For a weak laser excitation, as in optical molasses, we may assume that $\rho_{ee}(t'')-\rho_{gg}(t'')$ in the integrand in Eq. (5) does not vary much during the time interval $t'-t$ [13], and hence can be replaced by $\rho_{ee}(t')-\rho_{gg}(t')$ and pulled out of the integral. We then obtain

$$\begin{aligned}\langle\hat{\sigma}_{-}(t)\rangle e^{i\omega t}&=\langle\hat{\sigma}_{-}(t')\rangle e^{i\omega t'}e^{-(\gamma'/2+i\Delta)(t-t')}\\&\quad-\frac{i\Omega}{2(\gamma'/2+i\Delta')}[\rho_{ee}(t')-\rho_{gg}(t')]\\&\quad\times e^{-i\vec{k}\cdot\vec{v}\,t}(1-e^{-(\gamma'/2+i\Delta')(t-t')}),\end{aligned} \tag{6}$$

where $\Delta' \equiv \Delta - \vec{k}\cdot\vec{v}$. It is clear that the expectation value of the dipole operator $\hat{\sigma}_-$ at a later time $t+\tau$ (where $\tau \geqslant 0$) can simply be written in terms of $\langle \hat{\sigma}_-(t)\rangle$ by allowing $t \rightarrow t+\tau$ and $t' \rightarrow t$ in Eq. (6).

We now apply the quantum regression theorem [11,13] to calculate the expectation value of the two-time atomic dipole correlation $\hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau)$ in terms of the single-time operators given by Eq. 6. We obtain

$$\begin{aligned}\langle \hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau)\rangle e^{i\omega\tau} &= \rho_{ee}(t)e^{-(\gamma'/2+i\Delta)\tau} + \frac{i\Omega}{2(\gamma'/2+i\Delta')} \\ &\quad \times \rho^*{}_{eg}(t)e^{-i\vec{k}\cdot\vec{v}(t+\tau)}(1-e^{-(\gamma'/2+i\Delta')\tau}), \quad (7)\end{aligned}$$

where we have used $\langle \hat{\sigma}_+(t)\hat{\sigma}_{ee}(t)\rangle = 0$ (because $\langle g|e\rangle = 0$) and $\langle \hat{\sigma}_+(t)\hat{\sigma}_{gg}(t)\rangle = \langle \hat{\sigma}_+(t)\rangle$ (because $\langle g|g\rangle = 1$).

In the steady state ($t \rightarrow \infty$), we simply replace $\rho_{ee}(t)$ and $\rho_{eg}(t)$ in Eq. (7) with their steady-state values $\rho^{ss}_{ee}$ and $\rho^{ss}_{eg}$ which are readily found from Eqs. (4) by setting $\dot{\rho}_{ee}$ and $\dot{\rho}_{eg}$ equal to zero. Substituting these steady-state values for $\rho_{ee}$ and $\rho_{eg}$ in Eq. (7), we obtain

$$\begin{aligned}\langle \hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau)\rangle_{ss} &= \frac{\gamma}{\gamma'} n_{th} e^{-(\gamma'/2+i\omega_{eg})\tau} \\ &\quad + \frac{\frac{\gamma}{\gamma'}|\Omega|^2}{4\Delta'^2+\gamma'^2+2|\Omega|^2} e^{-i\vec{k}\cdot\vec{v}\,\tau} e^{-i\omega\tau}. \quad (8)\end{aligned}$$

We need to perform an average over the Maxwell-Boltzmann distribution of velocities in the cold-atom cloud. The first term on the right-hand side of Eq. (8) has no velocity dependence and remains unchanged. The velocity average of the second term is easily evaluated if we make the following two crude approximations.

(1) We ignore the velocity dependence of $\Delta'$ in the denominator, i.e., we put $\Delta' \approx \Delta$. This is roughly justified because while typical values of $\Delta$ for optical molasses range from one to several linewidths below resonance, the magnitude of $kv$ is just a fraction of the linewidth. For example, in the case of $^{85}$Rb, the linewidth is $\gamma/2\pi \approx 6$ MHz, and for molasses at a temperature of $T = 50\ \mu$K we can estimate $kv \approx (2\pi/\lambda)\sqrt{3k_BT/m}$ (where $\lambda = 780$ nm, $m$ is the mass number and $k_B$ is the Boltzmann's constant) to be about 0.7 MHz.

(2) We replace $\vec{k}\cdot\vec{v}$ in the exponential with simply $-kv$. This, again, is roughly justified because the dominant contribution to the radiated field comes from atoms that counterpropagate relative to the red-detuned laser beam and are Doppler-shifted closest to resonance. Even though, in this paper, we only consider one laser beam, real molasses typically have multiple laser beams irradiating the atomic sample from multiple directions. This means that no matter in which direction a trapped atom may be moving, it will likely be counterpropagating with respect to one of the laser beams and therefore be predominantly interacting with that beam.

Using the above two approximations, we have $\langle e^{ikv\tau}\rangle_v = \exp(-k^2\tau^2 k_BT/2m)$. Therefore, we finally obtain the following expression for the steady-state value of $\langle \hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau)\rangle$ which, from Eq. (3), is also the numerator of $g^{(1)}(\tau)$:

$$\begin{aligned}\langle \hat{\sigma}_+(t)\hat{\sigma}_-(t+\tau)\rangle_{ss} &= \frac{\gamma}{\gamma'}\Bigg[n_{th}e^{-\gamma'\tau/2}e^{-i\omega_{eg}\tau} \\ &\quad + \rho^{ss}{}_{ee,0}\exp\left(-k^2\tau^2\frac{k_BT}{2m}\right)e^{-i\omega\tau}\Bigg], \quad (9)\end{aligned}$$

where $\rho^{ss}{}_{ee,0} \equiv |\Omega|^2/(4\Delta^2+\gamma'^2+2|\Omega|^2)$. Note that if the factor $\gamma'^2$ in the denominator had instead been just $\gamma^2$, then $\rho^{ss}{}_{ee,0}$ would be nothing but the usual expression for the excited state fraction of a coherently excited two-level atom in the *absence* of radiation trapping.

Substituting Eq. (9) in Eq. (3), and using Eq. (2), we now have the expressions for the first- and second-order coherences of the light scattered from optical molasses, including the effect of radiation trapping. We present the implication of these results in the following section.

### C. First- and second-order coherences for the light scattered from a moving atom in the presence of radiation trapping

Equation (9) has an intuitively appealing justification. Keeping in mind that the Fourier transform of $g^{(1)}(\tau)$ is the frequency spectrum, we see that $g^{(1)}(\tau)$ has two terms; one oscillating at the atomic resonance frequency and the other at the driving frequency. Now, the emission spectrum, for a detuned excitation of strength such that $|\Omega| \sim \gamma$ or $I \sim I_s$, has two contributions [15]: (a) an elastic contribution, with a linewidth equal to that of the laser, centered at the laser frequency $\omega$; (b) a three-peaked inelastic contribution, comprising a central peak at $\omega$ and two side peaks located approximately at the atomic resonant frequency $\omega_{eg}$ and $2\omega - \omega_{eg}$, respectively [15], with linewidths comparable to the natural atomic linewidth. Note that the inelastic peak at $\omega$ is much smaller than the elastic contribution for weak excitation, while the emission at $2\omega - \omega_{eg}$ is much less likely than the $\omega_{eg}$ light to be reabsorbed in the cloud owing to the decrease of the absorption cross section $\sigma(\omega)$ with detuning. Therefore, in a crude sense, we may ignore the inelastic contribution at frequencies other than at $\omega_{eg}$. Hence, we find that the first term in Eq. (9), which depends on $n_{th}$ and arises solely from the contribution of radiation trapping to $g^{(1)}(\tau)$, is centered at the atomic resonance frequency $\omega_{eg}$. The second term describes the contribution to $g^{(1)}(\tau)$ from atomic emission arising predominantly from laser excitation (note, however, the dependence of this term on $n_{th}$ through $\gamma'$ in the denominator as mentioned earlier) and is hence located at the

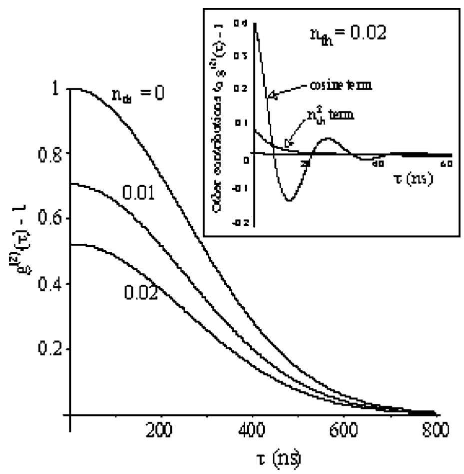


FIG. 2. The intensity correlation function $g^{(2)}(\tau)$ for $n_{th} = 0$, 0.01, and 0.02. The main plot shows just the contribution from the dominant first term in Eq. (11). The inset shows an expanded view of contributions at short time delays from the much smaller second and third terms in Eq. (11) to the correlation for $n_{th}$=0.02.

laser frequency $\omega$. This paragraph agrees with our previous remarks made just after Eqs. (2) and (3): we have modeled the atoms as weakly excited independent radiators that are being pumped separately by a coherent source at $\omega$ and by an incoherent background reservoir comprising fluorescent photons at $\omega_{eg}$.

From Eqs. (9) and (3), we have

$$g^{(1)}(\tau) = \frac{n_{th}e^{-\gamma'\tau/2}e^{-i\omega_{eg}\tau} + \rho^{ss}{}_{ee,0}\exp(-k^2\tau^2k_BT/2m)e^{-i\omega\tau}}{n_{th}+\rho^{ss}{}_{ee,0}}, \tag{10}$$

which, upon substitution in Eq. (2), yields the final result of our paper: the degree of second-order temporal coherence $g^{(2)}(\tau)$ for the light scattered from a moving atom in the presence of radiation trapping. We obtain

$$g^{(2)}(\tau) = 1 + \frac{1}{(n_{th}+\rho^{ss}{}_{ee,0})^2}[\rho^{ss2}{}_{ee,0}e^{-k^2\tau^2k_BT/m} + n_{th}^2e^{-\gamma'\tau} + 2n_{th}\rho^{ss}{}_{ee,0}e^{-\gamma'\tau/2}e^{-k^2\tau^2k_BT/2m}\cos\Delta\tau]. \tag{11}$$

It is clear that because $n_{th} \ll 1$, the dominant contribution from radiation trapping to $g^{(2)}(\tau)$ comes from the first term. Note that in the absence of radiation trapping the two terms that depend explcitly on $n_{th}$ vanish, and we have simply $g^{(2)}(\tau) = 1 + \exp(-k^2k_BT/m)$. In the presence of radiaiton trapping the prefactor for the exponential term is no longer unity, and depends on $n_{\mathrm{th}}$, as seen from Eq. (11) and from the definition of $\rho^{ss}{}_{ee0}$ given just after Eq. (9).

### D. Results and Discussion

In Fig. 2, we plot the dominant first term contribution to $g^{(2)}(\tau)$ [see Eq. (11)] as a function of the delay $\tau$ for three different values of $n_{th}$. The much smaller contributions from the remaining two terms in Eq. (11) are shown in the inset. We have used typical numbers for optical molasses of $^{85}$Rb atoms: $T = 50\ \mu$K, $I/I_s = 2$, $\Delta/\gamma = 2$. For the topmost plot, we put $n_{th} = 0$, i.e., we assume the density of trapped atoms to be so low that there is no radiation trapping. For the middle and lowest plots, we put $n_{th} = 0.01$ and $n_{th} = 0.02$, respectively, which from the definition of optical depth in Sec. III and from Eq. (1) correspond to an optical depth, number density of the cold atom cloud of about [0.15, $10^9/\mathrm{cm}^3$] and about [0.3, $2\times10^9/\mathrm{cm}^3$], respectively.

Note that if we add the contributions from all three terms in Eq. (11) we will obtain curves for $g^{(2)}(\tau)$ that always start from 2 at zero delay and fall to 1 for long delays, as we would expect for radiation from a collection of independent radiators [12]. However, for atomic samples of size larger than an optical wavelength one may expect the cosine oscillation to wash out owing to a random phase difference (corresponding to the random locations of atoms in the cloud) appearing between the incoherent and coherent pumping terms in Eq. (10). Of course, the small $n_{th}^2$ term still survives: an exponential that damps on a time scale determined essentially by the photon scattering rate. This appears as a small narrow peak superimposed on top of the main broad peak contributed by the $(\rho^{ss}{}_{ee,0})^2$ term which is an exponential with a width determined by the temperature of the atom sample. Thus, as radiation trapping increases one basically expects to measure a decrease in the coherence of the scattered light as shown in the main plots in Fig. 2.

In Ref. [8], it was shown that when effects of radiation trapping are absent, as in the topmost plot in Fig. 2, the width of the $g^{(2)}$ curve is determined by the Doppler broadening of the cold atom sample and therefore yields *in situ*, noninvasive information about the temperature of the cloud. In that work, the data suggested that the possible presence of radiation trapping may modify the intensity correlation function without measurably affecting the temperature of the sample (as measured by a standard time-of-flight technique). It is evident from the calculations here that small amounts of radiation trapping at moderate densities may cause substantial changes to the intensity correlation function. Note that the on-resonance optical depth of the atom sample in our calculations (~0.15–0.3) is significantly lower than the optical depth used in earlier investigations of radiation trapping in cold-atom clouds [1].

## IV. CONCLUSION

We have shown that the intensity correlation function $g^{(2)}(\tau)$ for the light scattered from optical molasses is extremely sensitive to the presence of radiation trapping. We have developed a physical, though admittedly rather crude, model of radiation trapping in optical molasses. The model incorporates into the optical Bloch equations the probability of reabsorption of photons emitted by the trapped atoms via the parameter $n_{th}$ [see Eq. (1)]. Our model includes the motion of the atoms in the laser-atom interaction but not in the interaction of the atoms with the trapped photons. While the full theory of radiation trapping in optical molasses is rather difficult, our simple model enables us to predict that there should be substantial changes in $g^{(2)}(\tau)$ even at optical depths over an order of magnitude lower than where effects of radiation trapping in cold atoms were previously reported. This model may also be useful for analyzing the coherent backscattering cone of light from cold-atom clouds [16].

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