The role of quantum jumps in the squeezing of resonance fluorescence from short-lived and long-lived atoms

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Abstract

Phase-sensitive squeezing in the resonance fluorescence of two-level atoms, that are coherently driven by a near-resonant laser field in free space, was observed recently (Lu *et al* 1998 *Phys. Rev. Lett.* **81** 3635). This was accomplished via homodyne detection at a phase near $\pm 45°$ relative to the driving field for strong off-resonant excitation of 'long-lived' atoms (where the atomic lifetime far exceeded the laser–atom interaction time, meaning that relaxation effects could be ignored). On the other hand, traditional theoretical predictions of phase-sensitive squeezing in the resonance fluorescence from two-level atoms have emphasized in- and out-of-phase (i.e., 0° and 90°) quadratures, and weak, on-resonant excitation of 'short-lived' atoms (where the observation time for laser–atom interaction far exceeded the natural atomic lifetime, meaning that relaxation effects dominate). Here, we calculate the probability of a delayed-coincidence detection in the interference field of a fluorescing dipole with a local oscillator (LO). We show that, despite the strikingly different conditions in which squeezing occurs in short- and long-lived atoms, squeezing in *both* cases can be shown to arise from a joint detection of two photons which are related by a quantum jump in the following way: the first photodetection precipitates a quantum jump of the atom to the ground state, and the second measures the mean amplitude of the fluorescent field subsequent to the quantum jump.

Phase-sensitive squeezing [1, 2] in the fluorescence of two-level atoms coherently driven by a near-resonant laser beam in free space is a well studied topic in quantum optics theory. However, a recent observation of squeezing spectra [3] has raised an intriguing question about squeezing in 'short-lived' versus 'long-lived' atoms. Traditional theoretical analyses of phase-sensitive squeezing [1, 2] have been made in the context of a 'short-lived' atom, where the laser–atom interaction time τ_0 was much greater than the natural atomic lifetime $1/\gamma$, meaning that relaxation effects dominate. On the other hand, the only observation to date of squeezing spectra [3] in phase-dependent resonance fluorescence was performed on 'long-lived' atoms, where the atomic lifetime far exceeded the interaction time, meaning that relaxation effects can be ignored. Further, past theoretical predictions [1, 2] of squeezing in short-lived atoms have focused on the case of weak on-resonant excitation and homodyne detection of the in- and out-of-phase (0° and 90° respectively) quadratures. But the observation of squeezing in long-lived atoms by Lu *et al* was performed at strong excitation with non-zero detuning via homodyne detection at a phase near $\pm 45°$ relative to the driving field. The following question then naturally arises: are the physical atomic processes that lead to phase-sensitive squeezing in short- and long-lived atoms one and the same, or are these processes different in some fundamental way?
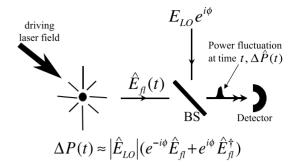
Here, we answer this question by using a simple physical picture of phase-dependent resonance fluorescence, developed originally by Lu *et al* in the context of long-lived atoms to help interpret the observed squeezing spectra. This picture incorporates quantum collapses into a Bloch vector model, and is based on a rigorous quantum calculation of phase-dependent spectra for long-lived atoms [4]. In this paper we extend the model by Lu *et al* to the analysis of squeezing in short-lived atoms. Specifically, we show that the phase-dependent two-time autocorrelation function for the fluorescent field emitted by a short-lived atom can be written in a form identical to that for a long-lived atom—a form which suggests a physically appealing interpretation of phase-dependent resonance fluorescence in terms of quantum jumps. We show that despite the strikingly different conditions in which squeezing occurs in short- and long-lived atoms, squeezing in *both* cases can be considered to arise from the following basic process: let us say the first detection, which defines the start of the correlation measurement, is that of a photon emitted by the induced atomic dipole. This detection immediately precipitates a quantum jump of the atom to the ground state, i.e., *conditions* the dipole in the ground state. The probability of detecting a second photon is proportional to the mean amplitude of the field emitted by the atomic dipole as it evolves in time starting from the ground state. The Fourier transform of this conditional temporal correlation is responsible for the phase-sensitive squeezing observed by Lu *et al* for long-lived atoms, as well as the squeezing predicted for short-lived atoms. This simple identification of the temporal process that leads to squeezing in the frequency spectra is by no means obvious from any previous analyses of squeezing. We further elucidate our interpretation by examining our results from the point of view of 'conditional homodyne detection', a seminal concept in quantum optics pioneered recently by Carmichael and co-workers [5, 6].

We begin by defining in section 1 below the relevant quantities for describing phase-dependent resonance fluorescence, namely the two-time autocorrelation function for the fluorescent field and its Fourier transform, which is closely related to the squeezing spectrum. In section 2 we briefly recall from [3] the main results of their calculation of the autocorrelation and the squeezing spectrum for the case of long-lived atoms. In section 3 we show how the results of previous workers on squeezing in short-lived atoms can be re-cast in the language of quantum jumps used by Lu *et al*. Finally, we state our conclusions.

1. Phase-dependent resonance fluorescence

Phase-dependent effects in resonance fluorescence are typically measured by mixing the fluorescent field $\hat{E}_{\mathrm{fl}}(t)$ (the caret denotes a quantum operator) with a strong local oscillator (LO) field $|E_{\mathrm{LO}}|e^{i\phi}$ having a controllable fixed phase ϕ relative to the driving field, and allowing the resultant field to be incident on a detector [3, 4]. For example, one such measurement scheme is shown in figure 1, where the fluorescent and LO fields are homodyned at a beamsplitter. The interference between the atom field and the LO causes fluctuations in the mean detected power $\Delta \hat{P}$ where

$$\Delta \hat{P}(t) \propto |E_{\mathrm{LO}}|(e^{-i\phi}\hat{E}_{\mathrm{fl}}(t) + e^{i\phi}\hat{E}_{\mathrm{fl}}^{\dagger}(t)). \qquad (1)$$

$$\Delta P(t) \approx |\hat{E}_{LO}|(e^{-i\phi}\hat{E}_{fl} + e^{i\phi}\hat{E}_{fl}^{\dagger})$$

Figure 1. Measurement of phase-dependent effects in resonance fluorescence by homodyne detection. Fluorescence from a coherently driven atom is mixed with a local oscillator field (LO) at a beamsplitter (BS). The LO is derived from the driving laser field.

Here, $\langle \Delta \hat{P} \rangle$ is proportional to the probability of a photodetection occurring at time t. The quantity $e^{-i\phi}\langle \hat{E}_{\mathrm{fl}}(t) \rangle + e^{i\phi}\langle \hat{E}_{\mathrm{fl}}^{\dagger}(t) \rangle$ is referred to as the quadrature field. Setting $\phi = 0$ ($\phi = 90°$) extracts the component of the quadrature field that is in phase (out of phase) with the LO. Using standard notation $\hat{\sigma}_{\pm}(t)$ for the atomic dipole raising/lowering operators of a two-level atom, we obtain $\hat{E}_{\mathrm{fl}}(t) \propto \hat{\sigma}_{+}(t)$ and $\hat{E}_{\mathrm{fl}}^{\dagger}(t) \propto \hat{\sigma}_{-}(t)$, where retardation effects from the atom to the detector are neglected. Thus, $\langle \Delta \hat{P} \rangle$ is proportional to the probability that the mean atomic dipole will radiate into the LO mode at time t.

There are two properties of the above interference in equation (1) that are relevant to our discussion of squeezing—the two-time phase-sensitive autocorrelation function $\langle :\Delta \hat{P}(t)\Delta \hat{P}(t + \tau): \rangle$ (where the notation ':· · · :' denotes time and normal ordering) and its Fourier transform, the phase-dependent power spectrum (which is closely related to the phase-sensitive squeezing spectrum denoted by $S_{\phi}(\omega)$).

In the following sections we investigate the two-time autocorrelation function and the squeezing spectrum for the cases of the long-lived and the short-lived atom.

2. The long-lived atom: phase-sensitive correlations and the squeezing spectrum

In this section we summarize the main results from [3] and [4]. We add an important qualifying remark to the physical picture of phase-dependent resonance fluorescence offered in these works by re-examining their results from the point of view of 'conditional' homodyne detection [5, 6].

2.1. The two-time autocorrelation function

The paper by Lu *et al* on the experimental observation [3] of the squeezing spectrum of long-lived atoms included a calculation of the phase-sensitive two-time autocorrelation function $\langle :\Delta \hat{P}(t)\Delta \hat{P}(t + \tau): \rangle$ for the conditions of the experiment, i.e., for strong off-resonant excitation. This calculation used operator Bloch vector equations [4] and included nonzero detuning. Relaxation effects owing to spontaneous emission were neglected because the radiative lifetime of the long-lived atom far exceeded the laser–atom interaction time τ_0. The calculation of the two-time autocorrelation function assumed the atom started in the ground state at $t = 0$ and involved the evaluation of the expectation value of the two-time correlation

functions $\hat{\sigma}_+(t)\hat{\sigma}_\pm(t+\tau)$ and $\hat{\sigma}_\pm(t+\tau)\hat{\sigma}_-(t)$. The result of this rigorous quantum calculation is reproduced below from [4]:

$$\langle:\Delta\hat{P}(t)\Delta\hat{P}(t+\tau):\rangle = \frac{\Delta}{2\Omega'}\sin 2\phi[\sin\Omega't(1-\cos\Omega'\tau)$$
$$+ (1-\cos\Omega't)\sin\Omega'\tau] + \cos^2\phi[\sin\Omega't\sin\Omega'\tau$$
$$+ (1-\cos\Omega't)\cos\Omega'\tau] + \sin^2\phi(1-\cos\Omega't). \qquad (2)$$

Here $\Omega' = \sqrt{\Omega^2 + \Delta^2}$ where Ω is the Rabi frequency and Δ is the laser detuning, and a common factor proportional to $\frac{\Omega^2}{4\Omega'^2}|E_{LO}|^2$ has been suppressed on the right-hand side of equation (2).

In order to emphasize the role of quantum jumps, the above rigorous result may be re-expressed as a sum of two terms [3]

$$\langle:\Delta\hat{P}(t)\Delta\hat{P}(t+\tau):\rangle = \langle\Delta\hat{P}(t)\rangle\langle\Delta\hat{P}(\tau)\rangle_{g@t} + \tfrac{1}{2}P_e(t)M(\tau). \qquad (3)$$
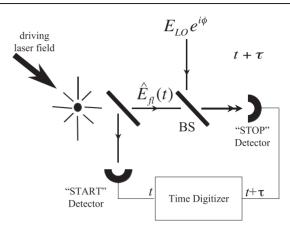
The subscript g to the first term on the right-hand side of equation (3) denotes a quantum jump at time t of the atomic dipole to the ground state. We now explain the meaning of the symbols in equation (3). Let us examine the first, i.e., the $\langle:\Delta\hat{P}(t)\Delta\hat{P}(t+\tau):\rangle$ term: for a strongly driven long-lived two-level atom the probability of a photodetection due to radiation from the mean dipole into the LO mode at time t is calculated [3] to be proportional to

$$\langle\Delta\hat{P}(t)\rangle = -\left(\frac{\Omega}{\Omega'}\frac{\sin\Omega't}{2}\cos\phi + \frac{\Omega\Delta}{\Omega'^2}\frac{1-\cos\Omega't}{2}\sin\phi\right). \qquad (4)$$

A key feature of a radiating quantum oscillator is that immediately following the detection of a radiated photon at time t the atom wavefunction *must collapse to the ground state*. Starting from the ground state at time t, the atom's mean dipole starts out at zero then evolves with time and radiates into the LO mode at a later time $t+\tau$ with probability $\langle\Delta\hat{P}(t+\tau)\rangle_{g@t} = \langle\Delta\hat{P}(\tau)\rangle$; i.e., the probability of detection of the next photon radiated by the mean dipole into the LO mode depends only on the time difference τ. Lu *et al* [3] showed that it is this quantum jump of the atomic dipole at t that leads to the observed phase-dependent squeezing in resonance fluorescence from long-lived atoms, as we shall see below when we write down the squeezing spectrum.

Next we briefly explain the second term in equation (3), i.e., the $\tfrac{1}{2}P_e(t)M(\tau)$ term: a complete explanation of this term is irrelevant for the purpose of this paper because, as was shown by Lu *et al* [3] and as we shall see below when we evaluate the squeezing spectrum, this term cannot give rise to squeezing. However, this term is very interesting in its own right. Briefly, this term describes a correlation between a quantum fluctuation in the driving field at time t and the fluorescent field at a time τ later. A thorough understanding of the fundamental implications of this correlation was lacking at the time of the work by Lu *et al* [3]. We defer a detailed study and measurement of this term to a forthcoming publication [7]. In this paper, we will merely state what the symbols $P_e(t)$ and $M(\tau)$ represent and show that squeezing cannot arise from them. $P_e(t)$ denotes the excited state probability for a strongly driven long-lived two-level atom that is initially in the ground state at $t = 0$:

$$P_e(t) = \frac{\Omega^2}{2\Omega'^2}(1-\cos\Omega t). \qquad (5)$$

Figure 2. Measurement of phase-dependent effects in resonance fluorescence by conditional homodyne detection. A detection in the START detector at t from the fluorescing atom triggers the collection of data on the interference between the fluorescent field and the LO field.

$M(\tau)$ describes the evolution of the dipole induced by the driving field quantum fluctuation at time t and is given [3] by $M(\tau) = \cos^2\phi\,\cos\Omega'\tau + \sin^2\phi[\frac{\Omega^2}{\Omega'^2} + \frac{\Delta^2}{\Omega'^2}\cos\Omega'\tau]$. The factor '1/2' that pre-multiplies $P_e(t)M(\tau)$ in equation (3) expresses the 50% likelihood that the randomly phased quantum fluctuation of the driving field is in the LO mode and is detected.

The important benefit of expressing the phase-sensitive two-time autocorrelation for long-lived atoms in the language of quantum jumps as embodied by equation (3) is this: when we calculate the squeezing spectrum in section 2.2, we find we can easily track the individual contributions made to the spectrum by each of the two terms in equation (3). We may, therefore, infer which term leads to phase-sensitive squeezing in resonance fluorescence.

However, an important clarifying remark about the physical picture based on equation (3) presented in this section needs to be made. Equation (2) is the result of a rigorous Heisenberg calculation. Since equation (3) is nothing but equation (2) with its terms re-arranged, equation (3) is rigorously correct. Nevertheless, the introduction of the notion of 'quantum jumps' in equation (3) is ad hoc, making equation (3) perhaps seem contrived. The problem is that in the experimental scheme outlined in figure 1 there is no way of knowing, in principle, when the atom makes a quantum jump. A measurement scheme, which lends itself naturally to a discussion of quantum jumps in phase-dependent resonance fluorescence, is that of 'conditional' homodyne detection, depicted in figure 2. In this case, the phase-sensitive measurement of the fluorescent field amplitude at time $t+\tau$ is initiated by the detection of a photon emitted by the atom alone at t [5, 6, 8]. Thus, in this scheme, a detection in the 'start' detector at t means unequivocally that the atom has made a quantum jump to the ground state. The second term in equation (3) vanishes, and the joint probability of detection of two photons separated by a delay τ is simply proportional to $\langle\Delta\hat{P}(\tau)\rangle_g$, the mean amplitude of the field after a photon detection (quantum jump) at t [8]. As we shall see in our calculation of the squeezing spectrum in the next section, this term leads to squeezing.

2.2. The squeezing spectrum

Let us denote the total interaction time of the atom with the driving laser field as τ_0. Clearly, the delay time τ between the two photodetections in equation (3) satisfies the inequality $0 \leqslant \tau \leqslant \tau_0$, and the initial photodetection time t satisfies the inequality $0 \leqslant t \leqslant \tau_0 - \tau$. To evaluate the frequency spectrum it is necessary to first average equation (3) over the range of initial times t. For long-lived atoms the radiative lifetime far exceeds the interaction time. In this case, the photon detected at time $t + \tau$ *is the next emitted photon* after the first one at t, i.e., the probability is negligible that there was a spontaneous emission between t and $t + \tau$ which was not detected. This situation is radically different from that of a short-lived atom where the radiative lifetime is far less than the interaction time, meaning that spontaneous emission is highly likely to occur between the two times t and $t + \tau$ at which photodetections are registered. Thus for long-lived atoms the squeezing spectrum is identical to the phase-sensitive frequency spectrum which is just the Fourier transform of the time-averaged two-time autocorrelation function:

$$S_\phi(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau \, \langle :\Delta\hat{P}(t)\Delta\hat{P}(t+\tau): \rangle_t e^{i\omega\tau}, \quad (6)$$

where $\langle \cdots \rangle_t$ denotes the average over initial times t. For strongly excited long-lived atoms $\Omega'\tau_0 \gg 1$, all terms in equation (3) containing $\sin \Omega' t$ or $\cos \Omega' t$ are eliminated by the average over t (typically many Rabi oscillations occur during the interaction time). Therefore, in this case, the squeezing spectrum $S_\phi(\omega)$ may be written as a sum of two terms, $S_{\Delta P}(\omega)$ and $S_M(\omega)$, arising from the $\langle \Delta\hat{P}(t) \rangle \langle \Delta\hat{P}(\tau) \rangle_g$ term and the $\frac{1}{2} P_e(t) M(\tau)$ respectively (see equation (3)):

$$
\begin{aligned}
S_{\Delta P}(\omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau \, \langle \Delta\hat{P}(t) \rangle_t \langle \Delta\hat{P}(\tau) \rangle \\
&= \left[\frac{\Delta^2}{\Omega'^2} \sin^2\phi \{2L(\omega\tau_0) - L(\delta\tau_0)\} \right. \\
&\quad \left. + \frac{\Delta}{\Omega'} \frac{\sin(\delta\tau_0) - \delta\tau_0}{(\delta\tau_0)^2} \sin 2\phi \right].
\end{aligned} \quad (7)
$$

and

$$
\begin{aligned}
S_M(\omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau \, \frac{1}{2} \langle P_e(t) \rangle_t M(\tau) e^{i\omega\tau} \\
&= \left[L(\delta\tau_0)\cos^2\phi + \left(\frac{\Delta^2}{\Omega'^2} L(\delta\tau_0) + 2\frac{\Omega^2}{\Omega'^2} L(\omega\tau_0) \right) \sin^2\phi \right],
\end{aligned} \quad (8)
$$

where $\langle \Delta\hat{P}(t) \rangle$ and $P_e(t)$ are given by equations (4) and (5) respectively. Here $\delta \equiv \omega - \Omega'$ and $L(x) \equiv \frac{\sin^2 x}{x^2}$. A common factor of $\frac{\Omega^2 \tau_0}{8\pi \Omega'^2}$ has been suppressed on the right-hand side of the final expressions for equations (7) and (8).

We may easily see from equation (8) that $S_M(\omega)$ is always positive, hence cannot yield any squeezing. On the other hand, equation (7) tells us that $S_{\Delta P}(\omega)$ may become negative for certain values of Ω and Δ. But we have to be careful: when we add $S_M(\omega)$ and $S_{\Delta P}(\omega)$ to obtain the squeezing spectrum $S(\omega)$, we see that the '$-L(\delta\tau_0)$' term in equation (7) is exactly cancelled by a corresponding term in equation (8), hence the squeezing does not arise from this term. In fact, the squeezing arises entirely from the $\sin 2\phi$ term. For $\Delta \sin 2\phi \geqslant 0$, the

$\sin\delta\tau_0 - \delta\tau_0$ term becomes negative. Maximum squeezing is obtained by setting $\phi = 45°$.[1] This picture of squeezing which emphasizes the role of a quantum jump was provided by Lu *et al* in their investigation of phase-dependent resonance fluorescence from strongly driven two-level long-lived atoms. In the next section we show that this picture of squeezing may be transferred to the case of weakly driven two-level short-lived atoms.

3. The short-lived atom: phase-sensitive correlations and the squeezing spectrum

In this section we re-examine traditional calculations of squeezing in short-lived atoms. We cast these results in the language of quantum jumps formulated in the previous section. This viewpoint leads us to an interesting insight into squeezing in resonance fluorescence that has not been explored in the literature previously.

3.1. The two-time autocorrelation function

In order to derive $\langle :\Delta\hat{P}(t)\Delta\hat{P}(t+\tau): \rangle$ in this case, we again start with the optical Bloch equations, but include relaxation. The relaxation is denoted by $1/\gamma$, the 1/e-decay time with which the atomic excited state decays exponentially in time. The interaction time far exceeds the relaxation time, i.e., $\tau_0 \gg 1/\gamma$. We set the detuning equal to zero because maximum squeezing in this case occurs for on-resonant excitation [1, 2]. Note that past theoretical treatments omitted to clearly articulate a role for the LO, opting instead to calculate phase-dependent fluorescent fluctuations in terms of a relative phase between the *radiated* field and the driving field, denoted by angle θ [1, 2]. This means that θ is different from ϕ which was defined in the previous section as the relative angle between the LO and the driving field. However, the role of θ and ϕ is the same: fluorescent fluctuations in phase (out of phase) with the driving field are calculated by setting $\theta = 0 (90°)$. After solving the coupled Bloch equations for the mean dipole and the mean inversion we use the quantum regression theorem to calculate the same two-time correlation functions for $\sigma_\pm$ as mentioned in the previous section for the case of the long-lived atom. In analogy to equation (2) we obtain

$$
\begin{aligned}
\langle :\Delta\hat{P}(t)\Delta\hat{P}(t+\tau): \rangle &= \{a_1(t)a_1(\tau) + b_1(t)a_3(\tau)\} e^{-2i\theta} \\
&\quad + a_1(t)a_1^*(\tau) + b_1(t)a_2(\tau) + \text{c.c.},
\end{aligned} \quad (9)
$$

where the a and b coefficients are

$$
\begin{aligned}
a_1(t) &= -i\frac{\Omega\gamma}{\gamma^2 + 2\Omega^2} \\
&\quad \times \left[1 - e^{-3\gamma t/4}\left(\cos\mu t - \left(\frac{4\Omega^2 - \gamma^2}{4\gamma\mu} \right) \sin\mu t \right) \right]
\end{aligned}
$$

$$a_{2,3}(t) = \frac{1}{2}e^{-\gamma t/2} \pm \frac{1}{8\mu}e^{-3\gamma t/4} (\gamma\sin\mu t + 4\mu\cos\mu t)$$

$$b_1(t) = \frac{\Omega^2}{\gamma^2 + 2\Omega^2}\left[1 - e^{-3\gamma t/4}\left(\cos\mu t + \frac{3\gamma}{\mu}\sin\mu t \right) \right], \quad (10)$$

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[1] However, note that the $\sin 2\phi$ term, and hence the squeezing for the long-lived atom, is zero if the detuning $\Delta = 0$. The inclusion of non-zero detuning reduces the likelihood of the atom being in the excited state, thus suppressing the contribution to the phase-sensitive spectrum from the $\frac{1}{2} P_e(t) M(\tau)$ term in equation (3).

where $\mu = \sqrt{\Omega^2 - \gamma^2/16}$. Note that for weak excitation such that $\Omega < \gamma/4$, equations (10) are expressed in terms of non-oscillatory hyperbolic functions instead of trigonometric.

While equations (9) and (10) have been calculated previously by other workers, one is led to a rather remarkable conclusion if we re-express these equations in the language of quantum jumps. We first note that equations (4) and (5) refer to the quadrature field amplitude emitted by, and the excited state probability of, an off-resonantly strongly excited long-lived atom, and the corresponding equations for an on-resonantly weakly excited *short*-lived atom are given by

$$\langle \Delta P(t) \rangle = a_1(t) e^{-i\theta} + \text{c.c.} \tag{11}$$

$$P_e(t) = b_1(t). \tag{12}$$

It is remarkable that using equations (10) and (12) we can re-write equation (9) in a form that is identical to equation (3).[2] This means that equation (3), which was originally written for long-lived atoms, *is true even for resonantly excited short-lived atoms*. This quantum jump viewpoint is especially suitable for interpreting the squeezing spectrum which we examine in the following section. Of course, the qualifying remarks at the end of section 2.2 regarding conditional homodyne detection also apply to the discussion here in sections 3.1 and 3.2.

3.2. The squeezing spectrum

The traditional definition of the squeezing spectrum is given as [9]

$$S_\phi(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau \, \langle :\Delta \hat{P}(t), \Delta \hat{P}(t+\tau): \rangle e^{i\omega\tau}, \tag{13}$$

where $\langle \hat{A}, \hat{B} \rangle \equiv \langle \hat{A}\hat{B} \rangle - \langle \hat{A} \rangle \langle \hat{B} \rangle$. Because of the inclusion of relaxation any transient oscillations damp out soon enough and the mean intensity radiated by the fluorescing atom settles into a 'steady state' quickly. If the interaction time includes many relaxation cycles, i.e. $\tau_0 \gg 1/\gamma$, then it is appropriate to evaluate the two-time correlation function in equation (13) in the 'long-time limit' $t \to \infty$. Therefore, inserting $t \to \infty$ in equation (3) and using the definition of $\langle \hat{A}, \hat{B} \rangle$ above, we obtain in the steady state

$$\langle :\Delta \hat{P}(t), \Delta \hat{P}(t+\tau): \rangle \stackrel{t \to \infty}{=} \frac{1}{2} P_e(\infty) M(\tau)$$
$$+ \langle \Delta \hat{P}(\infty) \rangle \langle \Delta \hat{P}(\tau) \rangle - \langle \Delta \hat{P}(\infty) \rangle \langle \Delta \hat{P}(\infty) \rangle, \tag{14}$$

where P_e and $\Delta \hat{P}$ are obtained from equations (10) and (12) by setting $t \to \infty$. It is only reasonable that in the steady state, the two-time correlation depends only upon the delay, not upon the initial time t—i.e., the fluctuating fluorescent field exhibits 'stationarity'. In passing, we note from equation (14) that including the 'comma' in the definition of the squeezing spectrum in equation (13) enables the subtraction of the time-independent term $\langle \Delta \hat{P}(\infty) \rangle \langle \Delta \hat{P}(\infty) \rangle$, which ensures the absence of delta-function-like contributions, an unwanted feature if one were examining squeezing.

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[2] For a resonantly excited short-lived atom,

$$M(\tau) = 4 \left[e^{-\gamma\tau/2} \cos^2\theta - e^{-3\gamma\tau/4} \left(\frac{\gamma}{4\mu} \sin\mu\tau - \cos\mu\tau \right) \sin^2\theta \right].$$

Substituting equation (14) in (13), we find that, in analogy with the long-lived case, the squeezing spectrum $S_\theta(\omega)$ can be expressed as a sum of two terms, $S_{\Delta P}(\omega)$ and $S_M(\omega)$, arising from the $(\langle \Delta \hat{P}(\infty) \rangle \langle \Delta \hat{P}(\tau) \rangle - \langle \Delta \hat{P}(\infty) \rangle \langle \Delta \hat{P}(\infty) \rangle)$ term and the $\frac{1}{2} P_e(\infty) M(\tau)$ term respectively:

$$S_{\Delta P}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau \left[\langle \Delta \hat{P}(\infty) \rangle \langle \Delta \hat{P}(\tau) \rangle \right.$$
$$\left. - \langle \Delta \hat{P}(\infty) \rangle \langle \Delta \hat{P}(\infty) \rangle \right]$$
$$= \frac{2(\Omega^2 - \gamma^2 - \omega^2)}{\left(\Omega^2 + \gamma^2/2 - \omega^2 \right)^2 + (\frac{3}{2}\gamma\omega)^2} \sin^2\theta, \tag{15}$$

and

$$S_M(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau \, \frac{1}{2} P_e(\infty) M(\tau) e^{i\omega\tau}$$
$$= \frac{1}{\omega^2 + \gamma^2/4} \cos^2\theta + \frac{2\Omega^2 + \gamma^2 + \omega^2}{\left(\Omega^2 + \gamma^2/2 - \omega^2 \right)^2 + (\frac{3}{2}\gamma\omega)^2} \sin^2\theta, \tag{16}$$

where we have used $\langle \Delta \hat{P}(\infty) \rangle \langle \Delta \hat{P}(\tau) \rangle - \langle \Delta \hat{P}(\infty) \rangle \langle \Delta \hat{P}(\infty) \rangle = -4a_1(\infty)\{a_1(\tau) - a_1(\infty)\} \sin^2\theta$ based on equations (10) and (12). A common factor of $\frac{2\gamma^2\Omega^2}{2\Omega^2+\gamma^2}$ has been suppressed on the right-hand side in the final expressions for equations (15) and (16).

We can easily see from equation (16) that $S_M(\omega)$ is always positive, hence cannot yield any squeezing. On the other hand, equation (15) tells us that $S_{\Delta P}(\omega)$ may become negative for certain values of Ω and γ. This negative value is maximum when $\sin^2\theta$ is maximum, i.e., when $\theta = 90°$. Adding $S_M(\omega)$ and $S_{\Delta P}(\omega)$ together we obtain the squeezing spectrum $S_\theta(\omega)$ [1, 2]. We see that maximum squeezing occurs at $\theta = 90°$ for weak excitation $\Omega^2 < \frac{\gamma^2}{4}$, and is contributed *entirely* by $S_{\Delta P}(\omega)$ (equation (15)). Note from section 2 for long-lived atoms that even though maximum squeezing occurred for strong, off-resonant excitation in the 45° quadratures, the entire squeezing came from the $S_{\Delta P}(\omega)$ term (equation (7)) in that case too.

We summarize this section on short-lived atoms by stating the central result of this paper. By analysing phase-dependent resonance fluorescence in the language of quantum jumps we find that, no matter whether we are dealing with off-resonantly excited long-lived atoms where relaxation is ignored, or with resonantly excited short-lived atoms where the relaxation γ needs to be included, squeezing may be considered to arise from a quantum jump to the ground state of the mean induced atomic dipole. This is an important physical insight into the process of phase-dependent resonance fluorescence which was being obscured by the math in traditional theoretical treatments (see equations (9) and (10), for instance).

In conclusion, we have re-examined the topic of phase-dependent resonance fluorescence with an emphasis on understanding the role played by quantum jumps. We have derived the two-time phase-sensitive autocorrelation function in a form that clearly displays the quantum jump process. This form of the correlation function was derived earlier for long-lived atoms [3], but not for short-lived atoms. We show that writing the correlation function in the form of equation (3) enables a physical interpretation of the role played by quantum jumps in the phenomenon of phase-sensitive

squeezing. In particular, by adopting this viewpoint, we find that, despite striking differences in the circumstances under which squeezing is observable in the resonance fluorescence from a long-lived and a short-lived atom, in both cases squeezing may be considered to arise from the quantum collapse to the ground state of the mean induced dipole.

Acknowledgments

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